我们运用了物理学、数学、材料化学、人文心理学等知识。

我们建立了一个关于磨损值的总模型，在它由简单变得复杂的过程中我们一步步解决了这一系列问题。

首先我们基于archard理论建立了最初的磨损值模型，也就是用阶梯的光滑程度与损失的体积来综合反映磨损程。我们又建立了人单次使用楼梯所造成的磨损值的公式。联系两者可以推导出一段时间内楼梯的人流量，这就解决了问题1：楼梯使用的频率问题。

由于人们在不同上下行方向在楼梯上的落脚点的不同性，我们提出了落点概率模型，并将此模型与上述单次磨损公式运用到原模型中。也就是说不同的行人上下方向对应的脚的落点是不同的，继而落点概率函数是不同的。而不同的概率函数将导致不同程度的磨损，继而反应到了磨损值模型中，这帮助我们判断问题2：使用楼梯时，是否有偏向某个特定的行走方向。

考虑到不同人数同时使用楼梯所对应的落点概率模型是不同的。于是我们改进了落点概率模型。显然不同的落点概率模型对应了不同磨损值模型。我们从这一点出发来进行问题3的判断：有多少人同时使用楼梯？（是两人还是单列行走）

后续附加的问题中我们首先考虑了问题7：在archard理论中涉及了有关不同材质对应的不同的比例常量，这是我们解决它的出发点。我们可以确定磨损值以及archard中其他的变量值，我们利用最小二乘法等算法可以得出最符合公式的比例常量。而对比这个比例常量和特定材质的比例系数就可以确定该楼梯使用的材料了。

关于问题5，6，8。我们将进行楼梯内部也就是非顶层的磨损研究来解决这些问题。

在楼梯内部，原始的模型已经无法很好的表现其磨损值了。于是我们从引入了Archie定律，添加到了磨损值模型中。archie定律反映了材料所有位置的空隙度与电阻率的关系，显然经过改进可以很好的表示磨损值。而电阻率又因为氧化等会随着时间进行变化。

\textbf{Assumption 1:} 楼梯表面的磨损程度只与楼梯的光滑程度、经摩擦损失的体积有关。而楼梯深层的磨损程度只与材料空隙率有关。这些因素已经可以很大程度的表现出磨损程度，同时，这不至于使得要考虑的因素太多而使得模型太过复杂且难以应用到实际。

\textbf{Assumption 2:} 每个人虽然每次踩过楼梯对它造成的磨损程度不一，但我们考虑的是大量踩踏次数下的情况。于是我们可以假设每次踩踏造成的磨损程度是一致的，等于其平均值。

\textbf{Assumption 3:}不同方向行走情况下，人在楼梯处的落脚点是不一样的，这将导致整体的落脚点不同。因为一般情况下我们在楼梯上行走时更多踩在楼梯右边。

\textbf{Assumption 4:}不同人数同时使用楼梯也会导致落脚点的不同。这是显然的：单人行走时人们更倾向走在楼梯中间，而两人同行时更倾向于两人分别踩在楼梯两边。

\textbf{Assumption 5:}在现场的考古学家们可以为我们提供所需的数据。因为我们所需的数据仅仅需要操作一些容易使用的仪器就可以测量出，同时这并不会对楼梯造成破坏。

\textbf{Assumption 6:}我们假设人每次行走在楼梯上造成的磨损程度是可以线性叠加的。因为每一步所造成的磨损与施加的力成正比，这可以简化我们的模型。

原来的ourwork

We have established a comprehensive model for wear value, addressing a series of problems step by step as the model evolved from simple to complex.

Initially, we developed the basic wear value model based on Archard's theory, which integrates the smoothness of the stairs and the volume of wear to reflect the wear process. We also derived a formula for the wear value caused by a single use of the stairs. By linking these two, we were able to derive the pedestrian traffic volume over a period of time, thus addressing Problem 1: the frequency of stair usage.

Due to the varying footfall points of individuals while ascending and descending the stairs, we proposed a footfall probability model and incorporated this model into the original wear value model, along with the previously established formula for single-use wear. In other words, the footfall points differ for pedestrians walking in different directions, leading to distinct probability functions. These differing probability functions result in varying levels of wear, which are then reflected in the wear value model. This helped us address Problem 2: whether there is a tendency for pedestrians to favor a particular direction when using the stairs.

Considering that the footfall probability model varies with the number of people using the stairs simultaneously, we improved the footfall probability model. Clearly, different footfall probability models correspond to different wear value models. This was used to tackle Problem 3: How many people use the stairs simultaneously? (Is it two people or are they walking in single file?)

For the subsequent additional issues, we first addressed Problem 7: In Archard's theory, the proportional constants for different materials are involved, which served as our starting point. By determining the wear value and other variables in Archard's formula, we employed algorithms such as least squares to derive the most suitable proportional constant. Comparing this proportional constant with the specific material's proportional coefficient allowed us to determine the material used for the stairs.

Regarding Problems 5, 6, 7 and 8, we will focus on the wear study of the internal sections of the stairs (i.e., below the top layer) to address these issues. In the internal sections, the original model is no longer sufficient to accurately represent wear value. Therefore, we introduced Archie's law into the wear value model. Archie's law reflects the relationship between porosity and resistivity in all parts of the material, and with improvements, it can effectively represent wear value. Moreover, resistivity changes over time due to factors such as oxidation, further enhancing the accuracy of the model

The following is a diagram illustrating the main theories used for the wear values of the staircase surface and interior.

楼梯是历史建筑中常见的结构元素，即使最坚固的石材也会随着时间磨损。这种磨损不仅是物理变化，还记录了使用历史和人类活动。因此，研究楼梯的磨损情况能为考古学家提供关键证据，帮助揭示建筑的历史演变。我们的任务是从这些痕迹中推断出具体的历史细节。

The staircase is a common structural element in historical buildings. Even the most durable stone materials will experience wear over time. This wear is not just a physical change, but also carries the history of use and human activity. Therefore, studying the wear of stairs provides crucial evidence for archaeologists and helps reveal the historical evolution of the building. Our task is to deduce specific historical details from these traces of wear.

首先我们考虑了楼梯表面的光滑程度与缺失的体积，基于**archard理论**建立了**原始磨损值模型。**在分析问题的过程中我们又推导出了**单次磨损公式，**根据此我们分析了楼梯的使用频率。

First, we considered the smoothness of the staircase surface and the volume of material lost, and based on **Archard’s theory**, we established the **original Wear value model.** During the analysis, we also derived the **single wear formula**, which allowed us to analyze the stair usage frequency.

然后我们考虑到人在上下楼梯时的落脚点不同，这让我们提出了**落点概率模型。**我们以**单次磨损公式**为桥梁，建立了磨损值模型与落点概率模型的联系。这帮助我们回答了使用楼梯特定方向的问题。而进一步分析同时几人使用楼梯对落点概率模型造成的影响后，我们改进了落点概率模型，很好的表述了几人同时使用楼梯的情况。

Considering the different landing points when people go up and down the stairs, we proposed the **impact probability model.** Taking the **single wear formula** as a bridge, we established the connection between the **Wear value model** and the **impact probability model.** This helps us answer the question of using the stairs in a specific direction. Further analysis of the impact of several people using the stairs simultaneously on the **impact probability model** led us to improve it, which well describes the situation of several people using the stairs at the same time.

然而研究并未停止，我们深度研究了Archard理论，利用其中的**比例常数K/H**得到了确定楼梯所用材料的思路。对于其余的问题，我们发现**原始磨损值模型**难以正确反映真实情况。于是我们转向研究楼梯内部的磨损程度。我们引入了**Archie定律**顺利构建了**内部磨损值模型**。最后我们以不同角度考虑**完整磨损值模型**，一步步揭示了楼梯中蕴藏的更多历史信息。

However, the research did not stop there. We delved deeply into **Archard's theory** and used the **proportional constant K/H** to obtain ideas for determining the materials used for the stairs. For the remaining questions, we found that the **original wear value model** could hardly correctly reflect the real situation. Therefore, we turned to studying the internal wear degree of the stairs. We introduced **Archie's law** to successfully build the **internal wear value model.** Finally, we considered the **complete wear value model** from different angles and gradually revealed more historical information hidden in the stairs.

最后我们对内部磨损值模型进行了灵敏度分析。我们研究了不同材料、**ρ/ρ0**在0~1.6内以及楼梯体积**V\_total**在0~10dm^3内对磨损值的变化程度。

Finally, we conducted a **sensitivity analysis** of the **internal wear value model.** We investigated the degree of change in wear values for different materials, **ρ/ρ0** within the range of **0-1.6**, and the stair volume **V\_total** within the range of **0-10 dm³**.

\begin{lstlisting}[language=Python]

import numpy as np

import matplotlib.pyplot as plt

from matplotlib.colors import LinearSegmentedColormap

d\_values\_wood = np.linspace(12, 20, 100)

d\_values\_concrete = np.linspace(6, 12, 100)

d\_values\_stone = np.linspace(0, 6, 100)

m\_values = np.array([1, 2, 3])

rho\_0 = 100

V\_total = 10

n = 1.5

r\_d\_m = lambda d, m: 0.1 \* m \* np.exp(0.1 \* d)

rho\_d\_m = lambda d, m: rho\_0 \* (1 + 0.05 \* m) \* np.exp(0.02 \* d)

def W\_with\_noise(d, m, noise\_factor=0.05):

r = r\_d\_m(d, m)

rho = rho\_d\_m(d, m)

W\_values = r \* V\_total \* (rho\_0 / rho) \*\* n

noise = np.random.normal(0, noise\_factor, size=d.shape) # Generate noise

W\_values += noise

W\_values = np.maximum.accumulate(W\_values)

return W\_values

cmap = LinearSegmentedColormap.from\_list("yellow\_green\_blue", ["yellow", "lightgreen", "lightblue", "blue"])

fig, ax = plt.subplots(figsize=(10, 6))

material\_names = {1: "Stage 1", 2: "Stage 2", 3: "Stage 3"}

for m, d\_values in zip(m\_values, [d\_values\_wood, d\_values\_stone, d\_values\_concrete]):

W\_values = W\_with\_noise(d\_values, m)

norm = plt.Normalize(d\_values.min(), d\_values.max())

sm = plt.cm.ScalarMappable(cmap=cmap, norm=norm)

for i in range(1, len(d\_values)):

ax.plot(d\_values[i-1:i+1], W\_values[i-1:i+1], color=sm.to\_rgba(d\_values[i])) # Gradient color for each segment

sm.set\_array([])

cbar = fig.colorbar(sm, ax=ax)

plt.title(r"Wear Value $W(d, m)$ for Different Stages", fontsize=14)

plt.xlabel("Depth (d)", fontsize=12)

plt.ylabel("Wear Value $W(d, m)$", fontsize=12)

plt.legend(title="Stages", fontsize=12)

plt.grid(False)

plt.show()

\end{lstlisting}